Physics 566

Problem Set #3

Due: Friday Sept. 24, 2010

Problem 1: Adiabatic rapid passage (10 Points)

Suppose we have an *inhomogeneously* broadened system - e.g. a system of two-level oscillators with a distribution of resonance energies such as a thermal gas (Doppler broadening) or with a distribution is a solid crystal due to local strain effects. How can we apply a π-pulse to send all of atoms to the excited state with high probability?

(a) Qualitatively suppose we apply a radiation field with a frequency well below resonance ($\Delta \ll \Omega$) and sweep the field slowly up through resonance, ending well above resonance ($\Delta \gg \Omega$), on a time scale much slower that the Rabi frequency $T \gg \Omega^{-1}$, but fast compared to spontaneous emission $T \ll \Gamma^{-1}$. Use the Bloch-sphere magnetic resonance picture to show that population in the ground state will "adiabatically" be transferred to the excited state.

(b) Quantitatively, sketch the eigenvalues of the two level atom in the RWA as a function of the frequency of the laser. Use the adiabatic theorem of quantum mechanics to explain the transfer of population from ground to excited, and the constraints on the time scales. Why does this carry the whole inhomogeneously broadened sample?

Problem 2: Light forces on atoms (10 Points)

Electromagnetic fields can exert forces on atoms. This is force can be dissipative (the basis of laser cooling) or conservative (the basis for optical trapping, such as optical lattices). Suppose we are given a monochromatic, uniformly polarized laser field. The most general form is: $\mathbf{E}(\mathbf{x},t) = \vec{\varepsilon}_L E_0(\mathbf{x})\cos(\omega_L t + \phi(\mathbf{x}))$. Interaction of this field with a two-level atom is described, in the rotating wave and electric dipole approximations by, the Hamiltonian,

$$\hat{H}_{AL}(\mathbf{R}) = -\frac{\hbar\Omega(\mathbf{R})}{2}\left(e^{-i\phi(\mathbf{R})}e^{-i\omega_L t}|e\rangle\langle g| + e^{i\phi(\mathbf{R})}e^{i\omega_L t}|g\rangle\langle e|\right),$$

where $\mathbf{R}$ is the center of mass position of the atom, and $\hbar\Omega(\mathbf{R}) = \langle e|\hat{\mathbf{d}}\cdot\vec{\varepsilon}_L|g\rangle E_0(\mathbf{R})$. Assuming the internal state of the atom relaxes to its steady state much faster than the atom moves, we can neglect the quantum mechanics of the atom's center of mass, and treat its motion as a classical point particle (this is know as the "semiclassical model"). The local force operator on the atom is then $\hat{\mathbf{F}} = -\nabla\hat{H}_{AL}(\mathbf{R})$.

(a) Under these condition show that the mean force on the atom is, $\mathbf{F} = \mathbf{F}_{diss} + \mathbf{F}_{react}$, where

$$\mathbf{F}_{diss} = \frac{1}{2}\hbar\,\mathrm{v(t)}\;\Omega(\mathbf{R})\nabla\phi(\mathbf{R}) \text{ is the "dissipative force" and}$$

$$\mathbf{F}_{react} = \frac{1}{2}\hbar u(t)\nabla\Omega(\mathbf{R}) \text{ is the "reactive force",}$$

with $u = 2\mathrm{Re}(\rho_{ge}^{(s)}\, e^{-i(\omega_L t+\phi(\mathbf{R}))})$ and $\mathrm{v} = 2\mathrm{Im}(\rho_{ge}^{(s)}\, e^{-i(\omega_L t+\phi(\mathbf{R}))})$ the components of the Bloch vector in the rotating frame (here with phase ϕ), and $\rho_{ge}^{(s)}$ in the Schrödinger picture Note, u and v satisfy the usual Bloch equations given in class.

(b) Show that is steady state, the rate at which that laser does work on the atom, averaged over an optical period is: $\left\langle \frac{dW}{dt} \right\rangle_{s.s} = -\frac{\hbar\Omega_0\omega_L}{2}\mathrm{v}_{s.s}$. If we define the steady-state rate of absorbing photons by the power scattering per $\hbar\omega_L$, use the steady-state solution to the Bloch equation to show that $\left\langle \frac{dN_{scat}}{dt} \right\rangle_{s.s.} = \Gamma\rho_{ee}^{s.s.}$. Interpret this result.

(c) For the case of a plane wave $\mathbf{E}(\mathbf{R},t) = \vec{\varepsilon}_L E_0\cos(\omega_L t - \mathbf{k}\cdot\mathbf{R})$, show that in steady-state:

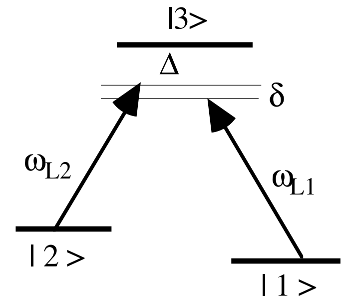
$$\mathbf{F}_{diss} = \hbar\mathbf{k}_L\left\langle \frac{dN_{scat}}{dt} \right\rangle_{s.s}.$$ This is known as "radiation pressure" - interpret.

(d) For the "reactive force" consider the case of weak saturation. Show that

$$\mathbf{F}_{react} = -\nabla U(\mathbf{R})$$

Problem 3: Raman-Rabi Flopping (20 Points)

Consider a three level atom in the so-called "lambda" configuration (because it looks like the Greek letter Λ):



Levels $|1\rangle$ and $|2\rangle$ are connected to level $|3\rangle$ on a dipole transition driven by lasers at frequencies ω_{L1} and ω_{L2} respectively. Laser-2 is detuned from resonance by Δ. Difference between the detunings of lasers 1 and 2 is $\delta = (\omega_{L1} - \omega_{L2}) - (E_2 - E_1)/\hbar$. The goal of this problem is to show that the lasers induce coherence between levels $|1\rangle$ and $|2\rangle$ and cause Rabi flopping population between them in an effective 2-level system. A transition between levels $|1\rangle$ and $|2\rangle$ is known as a "Raman transition".

(a) The Hamiltonian for this system (in the RWA) is $H = H_A + H_{AL}$ where

$$H_A = E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| + E_3|3\rangle\langle 3|,$$
$$H_{AL} = -\frac{\hbar\Omega_1}{2}\left(e^{-i\omega_{L1}t}|3\rangle\langle 1| + e^{i\omega_{L1}t}|1\rangle\langle 3|\right) - \frac{\hbar\Omega_2}{2}\left(e^{-i\omega_{L2}t}|3\rangle\langle 2| + e^{i\omega_{L2}t}|2\rangle\langle 3|\right).$$

where $\Omega_{1,2}$ are the two Rabi frequencies. Because there are two laser frequencies, the usual unitary transformation to the frame rotating at ω_L does not apply. However, one can perform a unitary transformation which makes H time independent.

Define a "new picture" via: $|\tilde{\psi}\rangle = U^\dagger|\psi\rangle$, $\tilde{H} = U^\dagger H U + i\hbar\frac{\partial U^\dagger}{\partial t}U$, with $U = \sum_{j=1}^{3} e^{-i\lambda_j t}|j\rangle\langle j|$

Show that for appropriate choice of λ_j we can transform H to,

$$\tilde{H} = \hbar\delta|1\rangle\langle 1| - \hbar\Delta|3\rangle\langle 3| - \frac{\hbar\Omega_1}{2}\left(|3\rangle\langle 1| + |1\rangle\langle 3|\right) - \frac{\hbar\Omega_2}{2}\left(|3\rangle\langle 2| + |2\rangle\langle 3|\right).$$

(b) Suppose that level-3 decays to level-1 at a rate Γ_{31} and level-2 with rate Γ_{32}. The dynamics of the density operator for this closed system is described by the master equation in the new picture:

$$\frac{d\tilde{\rho}}{dt} = \frac{1}{i\hbar}[\tilde{H},\tilde{\rho}] + \mathcal{L}_{relax}[\tilde{\rho}], \quad \text{where}$$

$$\begin{aligned}\mathcal{L}_{relax}[\tilde{\rho}] = &-\frac{\Gamma_{31}}{2}\left(|3\rangle\langle 3|\tilde{\rho} + \tilde{\rho}|3\rangle\langle 3| - 2|1\rangle\langle 3|\tilde{\rho}|3\rangle\langle 1|\right) \\ &-\frac{\Gamma_{32}}{2}\left(|3\rangle\langle 3|\tilde{\rho} + \tilde{\rho}|3\rangle\langle 3| - 2|2\rangle\langle 3|\tilde{\rho}|3\rangle\langle 2|\right)\end{aligned}.$$

Show that the matrix elements evolve according to:

$$\dot{\rho}_{11} = \Gamma_{31}\rho_{33} + i\frac{\Omega_1}{2}(\rho_{31} - \rho_{13}),$$
$$\dot{\rho}_{22} = \Gamma_{32}\rho_{33} + i\frac{\Omega_2}{2}(\rho_{32} - \rho_{23}),$$
$$\dot{\rho}_{33} = -\Gamma_3\rho_{33} + i\frac{\Omega_1}{2}(\rho_{13} - \rho_{31}) + i\frac{\Omega_2}{2}(\rho_{23} - \rho_{32}),$$
$$\dot{\rho}_{23} = -i(\Delta - i\Gamma_3/2)\rho_{23} + i\frac{\Omega_2}{2}(\rho_{33} - \rho_{22}) - i\frac{\Omega_1}{2}\rho_{21},$$
$$\dot{\rho}_{13} = -i(\delta + \Delta - i\Gamma_3/2)\rho_{13} + i\frac{\Omega_1}{2}(\rho_{33} - \rho_{11}) - i\frac{\Omega_2}{2}\rho_{12},$$
$$\dot{\rho}_{12} = -i\delta\,\rho_{12} + i\frac{\Omega_1}{2}\rho_{32} - i\frac{\Omega_2}{2}\rho_{13},$$

where $\Gamma_3 \equiv \Gamma_{31} + \Gamma_{32}$.

(b) These equations are of course rather complicated. We can solve for the dynamics in an important approximation. Suppose $\Delta >> \Omega, \Gamma, \delta$. Then the excited state populations and coherences with 3-1 and 3-2 relax to steady state, much more rapidly than the ground state populations and coherence 1-2. We can then *adiabatically eliminate these* variables. **Show that**

$$\rho_{33} = -\frac{\Omega_1}{\Gamma_3}\mathrm{Im}(\rho_{13}) - \frac{\Omega_2}{\Gamma_3}\mathrm{Im}(\rho_{23}), \quad \rho_{13} = \frac{1}{2\Delta - i\Gamma_3}(\Omega_1(\rho_{33} - \rho_{11}) - \Omega_2\rho_{12}),$$
$$\rho_{23} = \frac{1}{2\Delta - i\Gamma_3}(\Omega_2(\rho_{33} - \rho_{22}) - \Omega_1\rho_{21}).$$

(c) To lowest order in Ω, far off resonance show that

$$\rho_{33} \approx 0, \quad \rho_{13} \approx -\frac{1}{2\Delta}(\Omega_1\rho_{11} + \Omega_2\rho_{12}), \quad \rho_{23} \approx -\frac{1}{2\Delta}(\Omega_2\rho_{22} + \Omega_1\rho_{21}).$$

(d) Plug these back into the equations for the remaining matrix elements and show the remaining dynamics for level 1-2 has the form of Rabi flopping for a two level atom:

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -i\frac{\Omega_{eff}}{2}\left(\rho_{21} - \rho_{12}\right)$$
$$\dot{\rho}_{12} = -i\Delta_{eff}\ \rho_{12} - i\frac{\Omega_{eff}}{2}\left(\rho_{22} - \rho_{11}\right).$$

What is the effective Rabi frequency Ω_{eff}? What is the effective detuning Δ_{eff}? Interpret Ω_{eff} using second order perturbation theory.

(e) If we had retained the first nonvanishing term in Γ we would find the effective linewidth for this two-level system is $\gamma_{ram} = \left(\frac{s_1}{2} + \frac{s_2}{2}\right)\Gamma_3$, where $s_{1,2}$ are the saturation parameters for the two transitions. What is the condition that the Rabi flopping between 1-2 proceed coherently?

Epilogue: Coherent Raman transitions play an important role in laser spectroscopy and precision measurement. The ability to effectively reduce the effective linewidth by going further off-resonance is an important feature. There are many other interesting phenomena associated with atomic coherences induced in a three level system: Dark states, coherent population trapping, Electromagnetically Induced Transparency, and lasing without inversion, slow light, etc.. For further details, see Scully and Zubairy, Chapter 7. We study one such problem next.